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Inclined magnetic field effect on natural convective slip flow through a porouschannel with convective boundary

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--***ABSTRACT***---
In this paper, we consider the inclined magnetic field, Hallcurrent and convective boundary on laminar incompressiblenatural convection flow through a porous channel withslip conditions. The governing equations determined theflow regime is transformed into a system of ordinary differentialequations by utilizing suitable similarity transformations.Spectral Quasilinearization Method (SQLM) has beenapplying to solve the dimensionless governing equations;those were received by using similarity transformations fromthe system of governing partial differential equations withboundary conditions. The influence of emerging parameterson fluid flow velocities and temperature are presented graphically.

KEYWORDS:-*Inclined magnetic field, Hall current, Convective boundary, Porous medium, SQLM*

I. INTRODUCTION

Natural convection flow with heat transfer in a porous channel has an incredible significance in various fields. Natural convection has attracted a great deal of attention from researchers because of its presence both in nature and engineering applications. In nature, convection cells formed from air rising above sunlight-warmed land or water are a major feature of all-weather systems. The significance and developments of heat transfer have been addressed by many researchers [1, 2]. Hatami and Ganji [3] studied the natural convection of sodium alginate (SA) non-Newtonian nanofluid flow between two vertical flat plates by analytical and numerical methods. Recently, Sheremet et.al [4] presented the natural convection in an inclined cavity with time-periodic temperature boundary conditions using nanofluids: application in solar collectors. Most recently, Dogonchi et.al [5] studied the nature of magnetohydrodynamic natural convection of *Cu*-water nanofluid ina wavy cavity using CVFEM numerically.

An inclined magnetic field is a magnetic field with anonzero inclination, has gained the attention of many researchers [6, 7].Selimefendigil and Oztop [8] analyzed the natural convectionin a flexible sided triangular cavity with internal heatgeneration under the effect of an inclined magnetic field. Al-Rashed et.al [9] presented the effectsof external magnetic field inclination on laminar naturalconvection heat transfer in CNT-water nanofluid filledcavity. Recently, Atashafrooz et.al [10] discussed the interactioneffects of an inclined magnetic field and nanofluid onforced convection heat transfer and flow irreversibility in aduct with an abrupt contraction.

Makinde and Aziz [11] studied the boundary layer flowof a nanofluid past a stretching sheet with a convectiveboundary condition. Oyelakin et.al [12] presented the unsteadyCassonnanofluid flow over a stretching sheet withthermal radiation, convective, and slip boundary conditions.Khan et.al [13] analyzed heat transfer effects on carbon nanotubessuspended nanofluid flow in a channel with nonparallelwalls under the effect of velocity slip boundary condition:a numerical study. Recently, Ellahi et.al [14] investigatedthe effects of MHD and slip on heat transfer boundarylayer flow over a moving plate based on specific entropygeneration. Most recently, Chemetov et.al [15] presentedthe weak-strong uniqueness for fluid-rigid body interactionproblems with slip boundary conditions.

In this article, the free convection Navier slip flow ina porous channel under an inclined magnetic field, Hall effectsare investigated. A Spectral quasilinearization method isemployed to solve the system of equations. This methodwas suggested by Bellman et.al [16] as an implication ofthe Newton-Raphson method. Ahmad and his co-workers[17, 18] have been used the quasilinearization method forNeumann and mixed boundary value problems. [19, 20]have been elongated the significance of the quasilinearizationmethod to a broad variety of nonlinear BVP's and fractional differential equations and its applications. The exactnessand capability of the Spectral quasilinearization schemeare explained by [21].

II. FORMULATION OF THE PROBLEM

Consider a steady incompressible, laminar free convection flow along the porous channel with distance *2d* apart. A uniform external magnetic field B_0 is applied in the direction which makes an angle $\alpha$with the positive direction of the *x*-axis, temperatures and concentrations are considered as shown in Figure 1. As the boundaries are infinitely extended in the *x*-direction, without loss of generality, we considered that the physical parameters are functions of *y* only. The properties of the fluid are presumed to be constant except for density variations in the buoyancy force term. With the above assumptions and Boussinesq approximations, the governing equations for the flow are given by

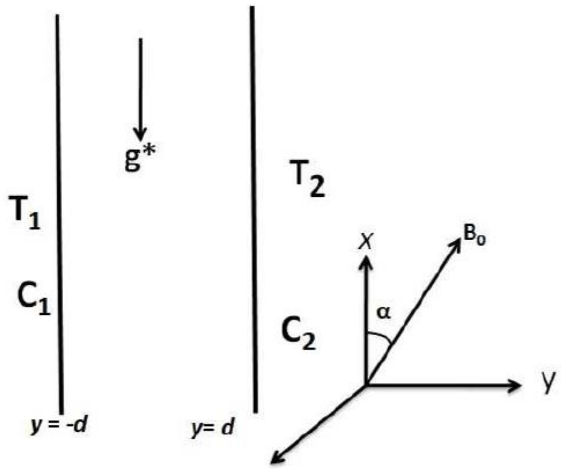

Fig. 1: Classical representation of the coordinate system.

$$\frac{\partial v}{\partial y} = 0 \Rightarrow v = v_0 \quad (1)$$

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} + \rho g^*[\beta_T(T - T_1)] - \frac{\sigma {B_0}^2 \cos\alpha}{1+m^2 cos^2\alpha}[u\cos\alpha - v_0 sin\alpha + mwcos^2\alpha] - \frac{\epsilon\mu}{K_f}u \ (2)$$

$$\rho v_0 \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma {B_0}^2 \cos\alpha}{1+m^2 cos^2\alpha}[mu\cos\alpha - mv_0 sin\alpha - w] - \frac{\epsilon\mu}{K_f}w \quad (3)$$

$$\rho C_p v_0 \frac{\partial T}{\partial y} = \frac{\sigma {B_0}^2}{1+m^2 cos^2\alpha}[(u\cos\alpha - v_0 sin\alpha)^2 + w^2 cos^2\alpha] + K_f \frac{\partial^2 T}{\partial y^2} + \mu[(\frac{\partial u}{\partial y})^2 + +(\frac{\partial w}{\partial y})^2] \quad (4)$$

With

$$y = -d: u = w = 0,\ k\frac{\partial T}{\partial y} = -h_1(T - T_1);\ y = d: u = w = 0,\ k\frac{\partial T}{\partial y} = -h_2(T - T_2)(5)$$

where *u, v* and *w* are the velocities in *x, y* and *z* respectively, μ is the coefficient of viscosity, g^* is the acceleration due to gravity, K_f is the thermal diffusion ratio, ρ is the density, C_p is the specific heat, $m = \eta_1 \sigma B_0$ is the Hall parameter, η_1= is the Hall factor, β_T is the coefficient of thermal expansion, $h_1$and $h_2$are the heat and mass transfer coefficients.

Introducing the following transformations

$$y = \eta d, u = \frac{\gamma Gr}{d}f, w = \frac{\gamma Gr}{d}g, T - T_1 = (T_2 - T_1)\theta(6)$$

Substitute in Eqs. (2) - (4), we obtain the governing dimensionless equations as

$$f'' - Ref' + \theta - \frac{Ha^2 cos\alpha}{1+m^2 cos^2\alpha}[fcos\alpha - \lambda sin\alpha + mgcos^2\alpha] - \frac{\epsilon}{Da}f = 0(7)$$

$$g'' - Reg' + \frac{Ha^2 cos^2\alpha}{1+m^2 cos^2\alpha}[mfcos\alpha - g - m\lambda sin\alpha] - \frac{\epsilon}{Da}g = 0\ (8)$$

$$\theta'' - RePr\theta' + \frac{BrHa^2}{1+m^2 cos^2\alpha}[(fcos\alpha - \lambda sin\alpha)^2 + g^2 cos^2\alpha] + Br\, Gr^2[(f')^2 + (g')^2] = 0(9)$$

With

$$f(-1) = g(-1) = \theta'(-1) - Bi_1(-1) = 0, \quad f(1) = g(1) = \theta'(1) - Bi_2(1) = -Bi_2 \quad (10)$$

where the primes indicate the differentiation concerningη, $Re = \frac{\rho v_0 d}{\square}$ is Reynolds number, $Gr = \frac{g^* \beta_T (T_2 - T_1) d^3}{\nu^2}$ is thermal Grashof number,$Pr = \frac{\mu C_P}{K_f}$ is Prandtl number, $Br = \frac{\mu \nu^2}{K_f d^2 (T_2 - T_1)}$ is Brinkman number, $Ha = dB_0 \sqrt{\frac{\sigma}{\upsilon}}$ is the magnetic parameter, $Bi_1 = \frac{h_1 d}{k}$, $Bi_2 = \frac{h_2 d}{k}$ are the Biot numbers, $\lambda = \frac{Re}{Gr}$, $Da = \frac{K_f}{d^2}$ is the Darcy number, and *m* is the Hall parameter.

III. DISCUSSION OF RESULTS

The flow Eqs. (7) - (9) subject to the boundary conditions(10) are nonlinear and coupled, hence thesystem ofequations is solved numerically using the Spectral quasilinearizationmethod as explained in the works of [24].
The influence of the Biot number (*Bi*) on $f(\eta)$, $g(\eta)$ and$\theta(\eta)$ can be shown in figures 2-4 at *Pr*=0.71,*Re*=2, *Gr*=2,ϵ=0.1, α=π/3, *Br*=3, *Da*=0.2, *Ha*=3, β_1=0.01 and β_2= 0.01.It is noted from figures 2-4 that the velocities and temperatureprofile decreases with anincrease in Biot number (*Bi*).

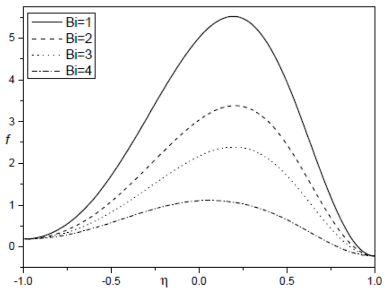

Fig 2. Effect of *Bi* on *x*- velocity

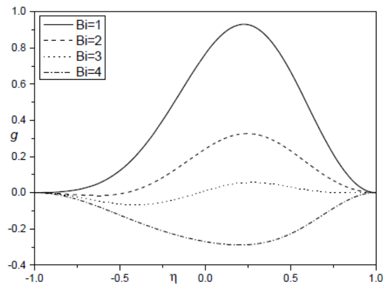

Fig 3. Effect of *Bi* on the cross-flow velocity

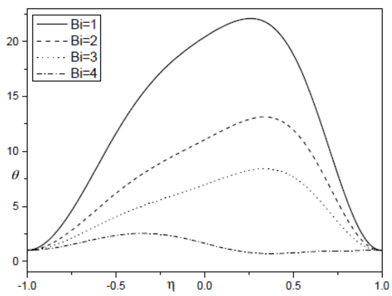

Fig 4. Effect of *Bi* on the temperature

The nature of $f(\eta)$, $g(\eta)$ and $\theta(\eta)$ at different valuesof *Ha*, *m*, *Da*, α,$\beta_1$and $\beta_2$are presented through Figs. 5 to22 by taking *Pr*, *Re*, *Gr*, ϵat 0.71, 2, 2, 0.1 respectively.
The impact of *m* on $f(\eta)$, $g(\eta)$, and $\theta(\eta)$ can be identifiedin Fig. 5-7 at α=π/3, *Br*=1, *Da*=0.2, *Ha*=2, *Bi*=5,β_1=0.1 and β_2= 0.1. It is noticed from Fig. 5-7 that the flow velocity, cross-flowvelocity, and temperature decrease with an increase in *m*.This is because of an inclined magnetic field. Here magneticfield applied with an angle α=π/3 and Hall effect producesin the direction of inclined plates, hence it cannot act as a dragon velocities. As enlighten above the Hall current producesadditional charge and which makes to decrease in temperatureof the fluid.

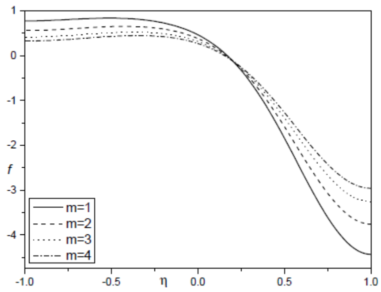

Fig 5. Effect of *m* on the flowvelocity

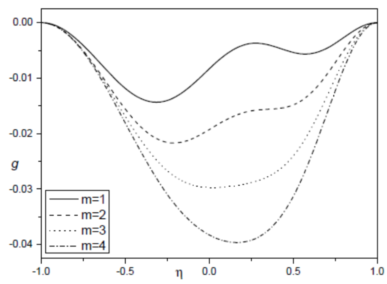

Fig 6. Effect of *m* on the cross flow velocity

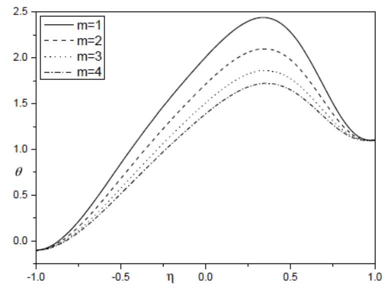

Fig 7. Effect of *m* on the temperature

Figure 8-10 presents the nature of *f*, *g*and θunder the influence of Darcy number (*Da*) when α=π/3, *Br*=2, *m*=4, *Ha*=2, *Bi*=5, β_1=0.1 and β_2= 0.1. It is noted from thesefigures that the flow velocities and temperature of the fluidincreases with an increase in *Da*.

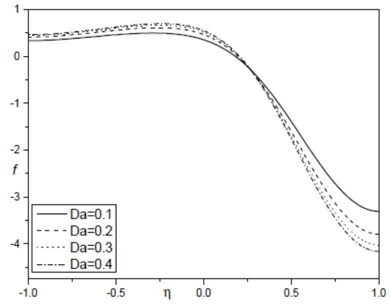

Fig 8. Effect of *Da* on the flow velocity

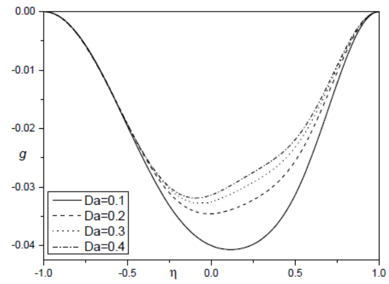

Fig 9. Effect of *Da* on the cross flow velocity

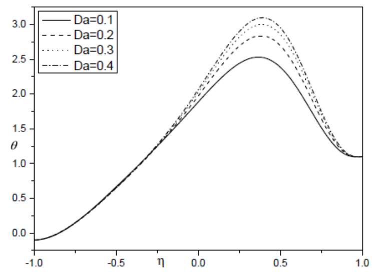

Fig 10. Effect of *Da* on temperature

Figure 11-13 presents the nature of *f*, *g* and θunder theinfluence of *Ha* when $\alpha=\pi/3$, $Br=1$, $Da=0.2$, $m=1$, $Bi=5$, $\beta_1=0.1$ and $\beta_2= 0.1$. It is found in Fig. 11 that as *Ha*enhancesthe *x*-velocity increases. Here the magneticfield has an inclination angle $\alpha> 0$ applied to an inclinedchannel generates the drag force. It is identified from Fig. 12that the cross-flow velocity increases as *Ha* increases.It can be shown from Figs. 13 that the temperature increaseswith the enhancement of the magnetic parameter. This is due to the reality that the magnetic field generates resistive power,which leads to an increase in the temperature.

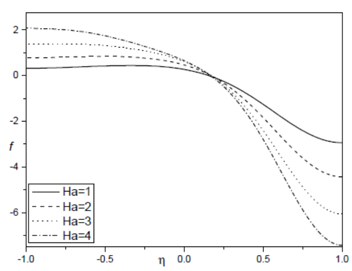

Fig 11. Influence of *Ha* on the flow velocity

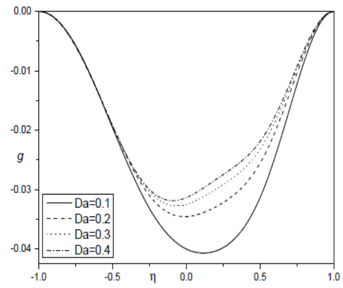

Fig 12. Influence of *Ha* on cross-flow velocity

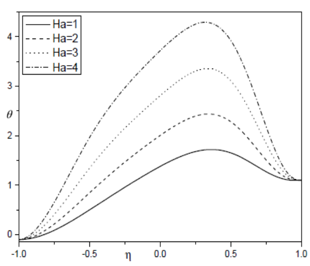

Fig 13. Influence of *Ha* on temperature

The impact of inclination angleα on*f*, *g*, and θcan benoted in Fig. 14-16 by fixing the other parameters at $Da=0.2$, $Br=2$, $m=2$, $Ha=2$, $Bi=5$, $\beta_1=0.1$ and $\beta_2= 0.1$. It is noticedfrom Fig. 14 and 16 that the flow velocity and dimensionlesstemperature increasesas αenhances. It is observed in figure 15 that the cross-flow velocity decreases as αincreases. This is due tothe reality that as an inclination angle of applied magnetic fieldchanges (angle of inclination increases) leads to the reductionin drag force will increase on the net flow.

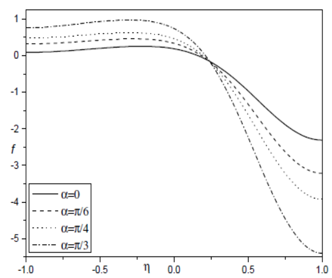

Fig 14. Effect of αon the flow velocity

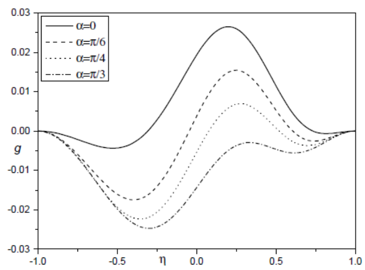

Fig 15. Effect of αon cross velocity

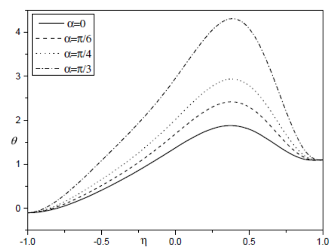

Fig 16. Effect of αon temperature

The effect of $\beta_1$on *f*, *g* and θcan be noted in Fig. 17-19 by fixing the other parameters at $\alpha=\pi/3$, $Br=1$, $Da=0.2$, $m=2$, $Bi=5$,$Ha=2$ and $\beta_2= 0.1$. It is observed from Fig. 19that the temperature of the fluid increases as an

$\beta_1$increases. It is noticed from Fig. 17 that the flow velocitydecreases as $\beta_1$increases. It is seen from Fig. 18 thatthe cross-flow velocity increases as $\beta_1$increases.

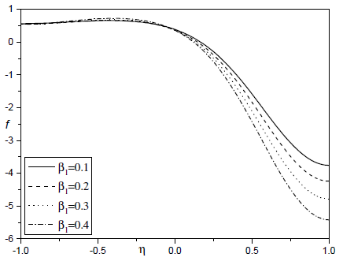

Fig 17. Effect of $\beta_1$on $f(\eta)$

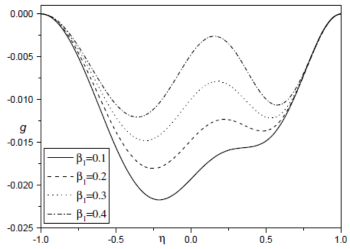

Fig 18. Effect of $\beta_1$on $g(\eta)$

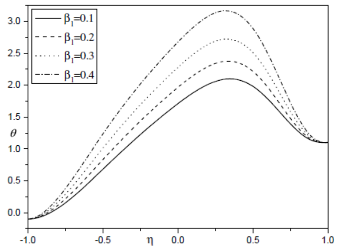

Fig 19. Effect of $\beta_1$on $\theta(\eta)$

The effect of$\beta_2$on *f*, *g* and θcan be noted in Fig. 20-22 by fixing the other parameters at $\alpha=\pi/3$, $Br=1$, $Da=0.2$, $m=2$, $Bi=5$, $Ha=2$ and $\beta_1=0.1$. It is noticed from Fig. 20that the flow velocity decreases as $\beta_2$increases. Itis observed from Fig. 22 that the temperature of the fluiddecreasesas an $\beta_2$increases. It is noticed from Fig. 21that the cross-flow velocity decreases as $\beta_2$increases.

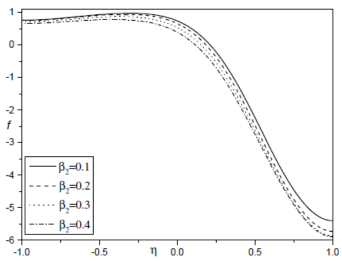

Fig 20. Effect of $\beta_2$on $f(\eta)$

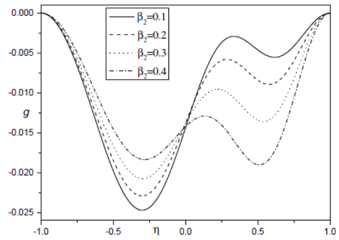

Fig 21. Effect of $\beta_2$on $g(\eta)$

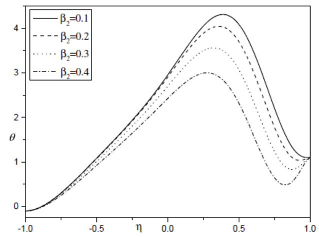

Fig 22. Effect of $\beta_2$on $\theta(\eta)$

IV. CONCLUSION

This article investigated the steady inclined magnetohydrodynamicnatural convective boundary condition in a porouschannel in the occurrence of Hall and Darcy effects. SpectralQuasilinearization Method is used to solve the dimensionlessgoverning equations. From this present study the mainfindings are listed as:

- The flow velocity profiles and temperature of the fluiddecreases with the increase of Hall parameter (m), Biotnumber (Bi) and slip parameter (β_2).
- Flow velocity and temperature of the fluid are increasesand cross-flow velocity profiles diminish with an enhancein inclination angle (α).
- The flow velocity profiles and temperature of the fluidincreases with the increase of Hartmann number (Ha)and Darcy number (Da).
- Flow velocity decreases and cross-flow velocity and temperatureare increases with the increase of slip parameter(β_1).

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